MATH 141: Midterm 1

Name: key

Directions:

- Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. If

$$f(x) = 1 - x^2 \qquad g(x) = 4x^3 - 2x^2 + 1 \qquad h(x) = \cos(x) \qquad j(x) = \frac{1}{x}$$

Evaluate, expand, and/or simplify the following:

(a) $h\left(\frac{13\pi}{6}\right) = \cos\left(\frac{13\pi}{6}\right) = \cos\left(2\pi + \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \boxed{\frac{\sqrt{3}}{2}}$

(b) $j(1) \cdot h(0) = \frac{1}{1} \cdot \cos(0) = 1 \cdot 1 = \boxed{1}$

(c) $f(x) \cdot g(x) = (1 - x^2) \cdot (4x^3 - 2x^2 + 1)$

$= 4x^3 - 2x^2 + 1 - x^2(4x^3 - 2x^2 + 1)$ dist law

$= 4x^3 - 2x^2 + 1 - 4x^5 + 2x^4 - x^2$ dist law

$= \boxed{-4x^5 + 2x^4 + 4x^3 - 3x^2 + 1}$

(d) $f(x+h) - f(x)$

You are subtracting into ≥ 2 terms!

$= 1 - (x+h)^2 - (1 - x^2)$

Subtraction is adding a factor of (-1). This is multiplication.

$= 1 - (x^2 + 2xh + h^2) - 1 + x^2$ dist law

$= 1 - x^2 - 2xh - h^2 - 1 + x^2$

$= -2xh - h^2$

$= \boxed{h(-2x - h)}$

2. Short answer questions:

(a) Write down the definition of the symbols $\lim_{x \to a} f(x) = L$.

As x approaches a, the heights $f(x)$ can be made as close as you want to L.

(b) True or false: We can simplify

$$\frac{(x+1)(x-2)-(x-1)(x+2)}{(x+1)^2(x-2)-(x-1)(x+2)}$$

by crossing out the $x+1$.

False. $(x+1)$ is not a global factor.

(c) If $f(x) = 2x^2$, evaluate $f(x+h)$ and fully expand + simplify.

$$f(x+h) = 2(x+h)^2 = 2(x^2 + 2xh + h^2)$$

$$= \boxed{2x^2 + 4xh + 2h^2}$$

(d) If $F(x) = \sqrt[3]{\sin(x^5)}$ find three functions f, g, h where $f \circ g \circ h = F$.

$f(x) = \sqrt[3]{x}$

$g(x) = \sin(x)$

$h(x) = x^5$

3. Suppose

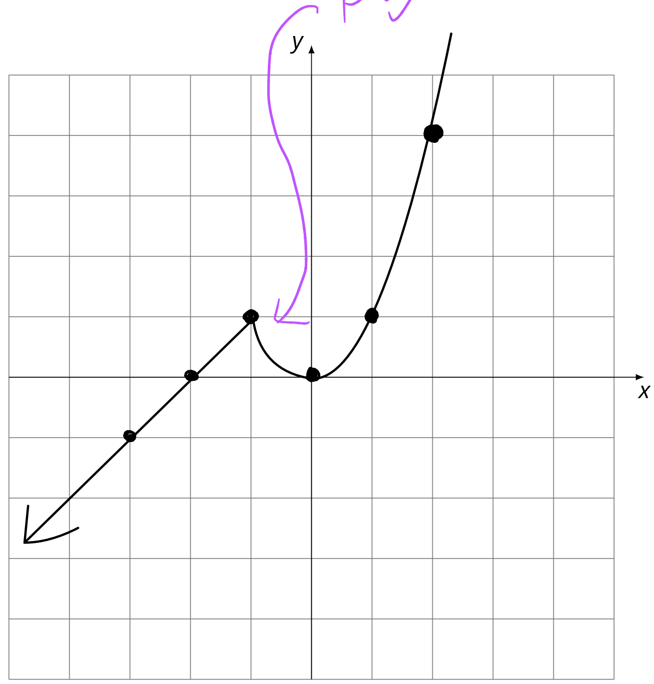
$$f(x) = \begin{cases} x+2 & x \leq -1 \\ x^2 & x > -1 \end{cases}$$

(a) What is $f(-1)$?

$f(-1) = -1 + 2 = 1.$

(b) Sketch a graph of $f(x)$.

x	$f(x)$
-3	$-3+2=-1$
-2	$-2+2=0$
-1	$-1+2=1$
0	$0^2=0$
1	$1^2=1$
2	$2^2=4$



plug in -1 into x^2 to get $(-1)^2=1$ so $(-1,1)$ is an open circle showing where the graph of x^2 starts.

(c) Does $\lim\limits_{x\to-1} f(x)$ exist? If it does, find the limit. If not, explain why.

Yes, from the graph $\lim\limits_{x\to-1^-} f(x) = 1 = \lim\limits_{x\to-1^+} f(x)$

so $\boxed{\lim\limits_{x\to-1} f(x) = 1}$

4. Perform the given instruction. Remember to use the relevant laws/properties and **fully simplify**.

   (a) Find the limit and simplify:

$$\lim_{h \to 0} \frac{(3+h)^2 - 9}{h}$$

Limit laws give $\frac{0}{0}$. $\lim\limits_{h \to 0}$ says create global factor of h and cancel. Denominator already has h. Focus on numerator as a pre-calc problem.

$$\lim_{h \to 0} \frac{(3+h)^2 - 9}{h} = \lim_{h \to 0} \frac{9 + 6h + h^2 - 9}{h}$$

$(A+B)^2$

$$= \lim_{h \to 0} \frac{6h + h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6+h)}{\cancel{h}}$$

GCF

$$= \lim_{h \to 0} [6 + h]$$

$$= \boxed{6}$$

(b) Find the limit and simplify:

$$\lim_{x\to 9} \frac{3-\sqrt{x}}{9x - x^2}$$

Try limit laws, you will get $\frac{0}{0}$.

Rationalize numerator:

Stop forgetting parentheses. $9x$ and x^2 are two terms. These two terms are being multiplied.

$$\lim_{x\to 9} \frac{\overbrace{3-\sqrt{x}}^{A\ -\ B}}{9x - x^2} \cdot \frac{\overbrace{3+\sqrt{x}}^{A\ +\ B}}{3+\sqrt{x}} = \lim_{x\to 9} \frac{9-x}{(9x-x^2)(3+\sqrt{x})}$$

$$= \lim_{x\to 9} \frac{\cancel{(9-x)}}{x\cancel{(9-x)}(3+\sqrt{x})}$$

GCF

$$= \lim_{x\to 9} \frac{1}{x(3+\sqrt{x})}$$

$$= \frac{\lim_{x\to 9} 1}{\left[\lim_{x\to 9} x\right] \cdot \left[\lim_{x\to 9} 3 + \sqrt{\lim_{x\to 9} x}\right]}$$

Limit laws.

$$= \frac{1}{9\cdot(3+\sqrt{9})}$$

$$= \frac{1}{9\cdot 6}$$

$$= \boxed{\frac{1}{54}}$$

(c) Simplify: $\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$ ← deal with numerator as a subproblem!

missing factor of x — missing factor of $(x+h)$

Beware! — this does NOT simplify as $-x+h$.

$$\frac{\frac{1}{(x+h)}-\frac{1}{x}}{h} = \frac{\frac{x}{x}\cdot\frac{1}{(x+h)}-\frac{1}{x}\cdot\frac{x+h}{x+h}}{h}$$

$$= \frac{\frac{x}{x(x+h)}-\frac{x+h}{x(x+h)}}{h}$$ fraction law 1

Again, subtracting ≥ 2 terms necessitates parenthesis.

$$= \frac{\frac{x-(x+h)}{x(x+h)}}{h}$$ fraction law 3

$$= \frac{\frac{x-x-h}{x(x+h)}}{h} = \frac{\frac{-h}{x(x+h)}}{h} = -\frac{\cancel{h}}{x(x+h)}\cdot\frac{1}{\cancel{h}} = \boxed{-\frac{1}{x(x+h)}}$$

(d) Expand and simplify: $\underbrace{3(x-1)^2}-\underbrace{(x+2)(x-3)2x}$

Two global terms, each of which are subproblems

$(A-B)^2$ — commutative law

$$3(x-1)^2-(x+2)(x-3)2x = 3(x^2-2x+1)-2x(x^2-x-6)$$

$$= 3x^2-6x+3-2x^3+2x^2+12x$$ dist law

$$= \boxed{-2x^3+5x^2+6x+3}$$

5. Draw the graph of a function which satisfies the following:

   (a) $f(-2) = 2$

   (b) $f(2) = -2$

   (c) $\lim_{x \to 2} f(x) = 1$

   (d) $\lim_{x \to -2^-} f(x) = 0$

   (e) $\lim_{x \to -2^+} f(x) = 1$

   (f) $\lim_{x \to 0} f(x) = \infty$